A PLANE SPECIMEN IN THE PERIODIC HEATING METHOD. MEASUREMENT OF HEAT CAPACITY

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We have undertaken a theoretical validation of the methods used to measure the thermophysical properties of materials by a method of periodic heating in the case of a disk specimen and a circular modulated flow of heat. We take into consideration the exchange of heat at all free surfaces of the specimen. We propose an expression to calculate the heat capacity of the material. The results derived in this study are compared against those yielded by the theory for the method of plane temperature waves.

An urgent problem at the present time involves research into the thermophysical properties of materials at high temperatures. Among the various methods used to determine the corresponding thermophysical characteristics we should take note of the pulse methods [1], as well as of the method of periodic heating [2] in which the parameters of the temperature wave are usually evaluated. In this case, increasingly widespread is the so-called method of plane temperature waves that is convenient from the standpoint of carrying out studies at high temperatures. The existing theory behind this method [3] is based on a one-dimensional model which, however, inadequately describes the true situation.

Individual aspects in the analysis of a two-dimensional model in the method of plane temperature waves had been dealt with earlier [4] but no consideration was given here to the full extent to which heat exchange affects the parameters of the temperature wave. Such consideration was undertaken in [5], but only for cases of a spot source and for a uniform form flow of heat acting over the entire surface of a specimen.

Another model has been proposed [6], and this more adequately describes the true physical situation that arises during the course of these experiments [7]. It thus became possible to develop an appropriate method to measure the thermophysical properties of materials, thereby to take into consideration the limitations imposed on the dimensions of the heat flow and the specimen and, consequently, to account for the distortions in the isothermal surfaces that arise in this case as a consequence of the lateral heat exchange and the nonuniformity in the flow of heat. In that same reference the authors made an attempt to determine the boundaries of applicability for the method of plane temperature waves in the study of the thermal diffusivity of solid materials.

The purpose of the present study is to provide a theoretical foundation for the method of measuring the thermophysical properties of materials, in particular the heat capacity.

As is well known, the method of temperature waves is complex and allows us to determine not only the coefficient of thermal diffusivity, but the heat capacity of materials as well [2]. In order to accomplish such measurements we must evaluate not only the phase but the amplitude of the temperature wave.

Let us briefly recall the formulation of the problem for the calculation of the temperature-wave parameters as presented in [6].

A disk specimen of radius R and thickness L, positioned in a chamber with inert gas, is subjected to the action of a modulated laser heat flow $q = q_0 \exp(i\omega t)$, which impinges on the center of one of the plane surfaces of the specimen, illuminating a spot of radius b. Here we investigate the parameters which are generated within the specimen with respect to the temperature wave under the assumption that its variable component is small with respect to the constant. As a result of the solution of this problem, with consideration of the exchange of heat at all of the surfaces of the disk specimen (through the introduction of corresponding Biot numbers) we derive an expression for the complex amplitude of the temperature wave

$$\Theta(r,\ z) = \frac{2q_0 bL^2 D}{\lambda R^2}, \tag{1}$$

where r and z are the instantaneous coordinates for the problem, directed perpendicular and parallel to the incident heat flow, calculated from the central point of the surface on which this flow of heat acts;

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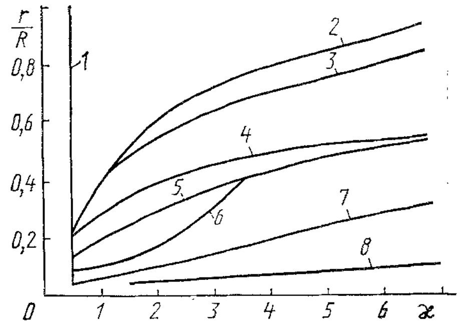

Fig. 1

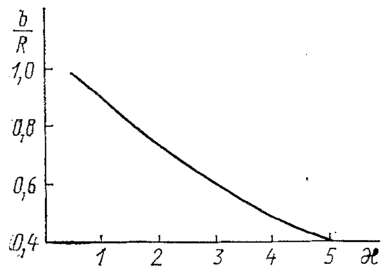

Fig. 2

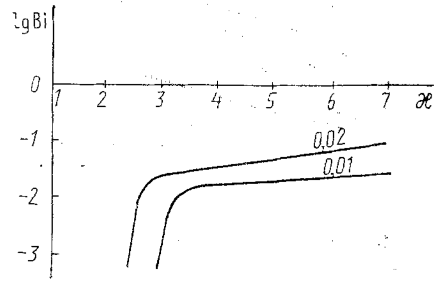

Fig. 3

Fig. 1. Boundaries of the $r/R(\kappa)$ regions for various values of Bi and b/R in which are satisfied $[|\Theta(r/R, \mathrm{Bi})| - |\Theta(0, \mathrm{Bi})|]/|\Theta(0, \mathrm{Bi})| < 0.01$: 1) Bi = 0-0.01, b/R = 1.0; 2) 0.2 and 1.0; 3) 1.0 and 1.0; 4) 2.0 and 0.8; 5) 1.0 and 0.8; 6) 0-0.2 and 0.8; 7) 0-2.0 and 0.6; 8) 0-2.0 and 0.4.

Fig. 2. Boundary of the region $b/R(\kappa)$ for Bi = 0-2.0; R/L = 5; r = 0, in which is satisfied $[|\Theta(0, \mathrm{Bi})| - |\Theta_I(\mathrm{Bi})|]/|\Theta_I(\mathrm{Bi})| < 0.01$.

Fig. 3. Boundaries of the log $\mathrm{Bi}(\kappa)$ regions for b/R = 0.6, R/L = 5, r = 0, in which is satisfied $[|(0, \mathrm{Bi})| - |\Theta_I(\mathrm{Bi} = 0)|]/|\Theta_I(\mathrm{Bi} = 0)| < 0.01, 0.02$.

$$D = \sum_{n=1}^{\infty} \frac{\mu_n J_1\left(\mu_n \frac{b}{L}\right) J_0\left(\mu_n \frac{r}{L}\right)\left[\rho_n \operatorname{ch}\left\{\rho_n\left(\frac{z}{L}-1\right)\right\} - \mathrm{Bi}_2 \operatorname{sh}\left\{\rho_n\left(\frac{z}{L}-1\right)\right\}\right]}{[\mathrm{Bi}_3^2 + \mu_n^2] J_0^2\left(\mu_n \frac{R}{L}\right)[(\mathrm{Bi}_1 \mathrm{Bi}_2 + \rho_n^2) \operatorname{sh} \rho_n + \rho_n (\mathrm{Bi}_1 + \mathrm{Bi}_2) \operatorname{ch} \rho_n]};$$

μ_n is the root of the equation $\mu_n J_1\left(\mu_n \frac{R}{L}\right) - \mathrm{Bi}_3 J_0\left(\mu_n \frac{R}{L}\right) = 0$; J_0, J_1 are Bessel functions; $\rho_n^2 = \mu_n^2 + i\kappa^2$, $\kappa^2 = (\omega/a)L^2$; $\mathrm{Bi}_{1,2,3}$ are the Biot numbers, respectively, for the surface affected by the heat flow, the opposite surface, and the side surfaces.

The expression for the amplitude of temperature fluctuations in the specimen

$$|\Theta| = \sqrt{(\mathrm{Re}\,\Theta)^2 + (\mathrm{Im}\,\Theta)^2} \tag{2}$$

allows us to analyze the applicability of the plane temperature wave method for measurement of the specific heat capacity C_p of the material making up the specimen, utilizing its relationship to the amplitude of the temperature fluctuations (2):

$$C_p = \frac{2PLF(\varkappa)}{M|\Theta| b\omega}, \tag{3}$$

where P = kQ; k is the absorption factor for the substance of the specimen; Q is the laser power;

$$F(\varkappa) = \varkappa^2 \sqrt{(\mathrm{Re}\,D)^2 + (\mathrm{Im}\,D)^2}.$$

With this purpose in mind, a computer was used to obtain the family of curves representing the relationship between the amplitude of $|\Theta|$ and the dimensionless parameter κ for various values of the Biot numbers (under the conditions that $Bi_1 = Bi_2 = Bi_3 = Bi$) Bi = 0-2.0 and relationships b/R = 0.2-1.0 at the surface of the disk specimen, opposite to the surface which is affected by the heat flow (here z = L).

We used the derived data to study the region of identical temperature-wave amplitudes, whose boundaries for the case R/L = 5 are shown in Fig. 1. To the right of these lines we have $[|\Theta(r/R, Bi)| - |\Theta(0, Bi)|]/|(0, Bi)| < 0.01$, i.e., we can neglect the change in the values of $|\Theta|$ in deviation from the central point with an error of 1%. Figures 2 and 3 illustrate the possibility of utilizing the results yielded by the one-dimensional theory in connection with the problem under consideration. These regions are situated to the right of the indicated boundary lines.

The cited results allow us to draw the following conclusions: 1) the region of identical amplitude on the flat disk surface of the specimen, opposite to the surface affected by the flow of heat, is smaller than the diameter of the illuminated spot; 2) approximation of the method of plane temperature waves remains valid only in the relatively small region of changes in the parameters Bi and b/R when $\kappa > 2$; 3) to calculate the heat capacity of the material of the specimen in the general case calls for utilization of expression (3) in conjunction with (1) and (2).

This theory serves as the basis for the development of a method of complex measurement for thermophysical properties of materials.

NOTATION

λ, coefficient of thermal conductivity for the specimen, W/(m·K); a, coefficient of thermal diffusivity for the specimen, m^2/sec; C_p, specific heat capacity of the specimen, J/(kg·K); L, specimen thickness, m; R, specimen radius, m; b, radius of light spot on surface of specimen, m; q, heat flux density, W/m^2; ω, modulation frequency for heat flow, rad/sec; Bi, Biot number; M, specimen mass, kg; P, specimen absorption power, W; Θ, complex amplitude of temperature wave, K; $|\Theta_I|$, amplitude of temperature fluctuations, calculated by the method of plane temperature waves for the same value of q.

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